

SHARP

ADVANCED D.A.L.



SCIENTIFIC CALCULATOR TEACHER'S GUIDE

EL-531RH

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Introduction

The use of calculators as a classroom teaching tool is becoming more and more popular. Contrary to the belief that their use encourages dependency and inhibits the development of mental skills, research has proven that calculators are highly unlikely to harm achievement in mathematics and using them can actually improve the students' performance and attitude.* Calculators allow students to quickly generate large amounts of data from which patterns can be spotted, and predictions can be made and tested. This is an important aspect of the development of mental methods of calculation. Therefore, priority must be given to create new ways to exploit the potential of the calculator as an effective learning tool in the classroom.

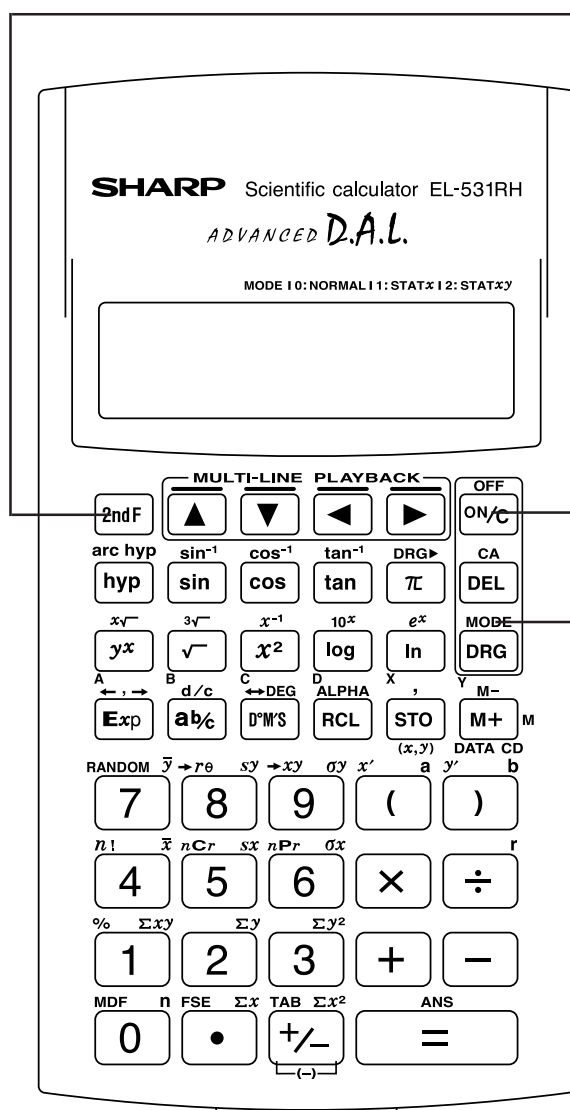
This Teacher's Guide presents several classroom activities that make use of Sharp scientific calculators. The purpose of these activities is not to introduce the calculator as a device to relieve the burden of performing difficult calculations, but rather to develop the students' understanding of mathematical concepts and explore areas of mathematics that would otherwise be inaccessible. Mental methods should always be considered as a first resort when tackling calculations introduced in these activities. The development of trial and improvement methods are supported by the activities as well. We hope you will find them interesting and useful for reinforcing your students' understanding of mathematical concepts.

* Mike Askew & Dylan Williams (1995) Recent Research in Mathematics Education HMSO

How to Operate

≈Read Before Using≈

1. KEY LAYOUT



2nd function key

Pressing this key will enable the functions written in yellow above the calculator buttons.

ON/C, OFF key

Direct function

ON/C
<Power on>

2nd function

2ndF **OFF**
<Power off>

Written in yellow above the ON/C key

Mode key

This calculator can operate in three different modes as follows.

<Example>

[Normal mode]

2ndF **MODE** **0**

•Mode = 0; normal mode for performing normal arithmetic and function calculations.

[STAT-1 mode]

2ndF **MODE** **1**

•Mode = 1; STAT-1 mode for performing 1-variable statistical calculations.

[STAT-2 mode]

2ndF **MODE** **2**

•Mode = 2; STAT-2 mode for performing 2-variable statistical calculations.

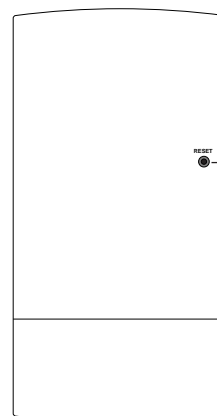
2. RESET SWITCH



If the calculator fails to operate normally, press the reset switch on the back to reinitialise the unit. The display format and calculation mode will return to their initial settings.

NOTE:

Pressing the reset switch will erase any data stored in memory.



Reset switch

Number Bowling

Junior high school

Objective

Read whole numbers and understand that the position of a digit signifies its value.
Understand and use the concept of place value in whole numbers.

Explanation of the activity

Think of a 3-digit number and enter it into your calculator.
Pretend each digit is a "bowling pin."
Knock down each pin one at a time, so that your calculator display shows 0.

A: Using subtraction

B: Using addition

Using the calculator

Calculator functions used: Subtraction, addition, last answer memory

A: Using subtraction

Press the following buttons and then start operation.

ON/C **2ndF** **MODE** **0**

(1) Enter a 3-digit number.

638 **=**

638= ^{DEG} 638.

(2) Knock down one digit, or "pin"; i.e. change the last digit to a 0.

- 8 **=**

ANS-8= ^{DEG} 630.

(3) Knock down the next pin; i.e. change the tens column digit to 0.

- 30 **=**

ANS-30= ^{DEG} 600.

(4) Knock down the pin of the hundreds column.

- 600 **=**

ANS-600= ^{DEG} 0.

B: Using addition

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

(1) Enter a 3-digit number.

638 [=]

638= DEG 638.

(2) Knock down one digit, or pin; i.e. change the last digit to a 0, except this time, do so by adding a number to the last digit to make it 0.

[+] 2 [=]

ANS+2= DEG 640.

(3) Knock down the next pin; i.e. change the tens column digit to 0.

[+] 60 [=]

ANS+60= DEG 700.

(4) Knock down the pin of the hundreds column.

[+] 300 [=]

ANS+300= DEG 1000.

..... Using the activity in the classroom

This activity is a good game for students to play in pairs.

One student enters a number in the calculator, and the other student has to knock each digit, or "pin," down.

Example:

$$638 - 8 = 630$$

$$630 - 30 = 600$$

$$600 - 600 = 0$$

..... Points for students to discuss

It is important for students to talk about what they are doing and use the appropriate language, for example: "six hundred and thirty, minus thirty, equals six hundred." Students should be challenged to vary the starting point; i.e. sometimes starting with the hundreds digit and sometimes with the tens digit.

Further Ideas

- Play the game using 2-, 4-, or 5-digit numbers according to the ability of the students.

Objective

Develop a variety of mental methods of computation.
 Develop the use of the four operations to solve problems.
 Use sequence methods of computation when appropriate to a problem.
 Estimate and approximate solutions to problems.

Explanation of the activity

Use the calculator to generate a 3-digit random number.
 The aim is to get the calculator to display the number 1.
 Players can use any of the numbers 1 – 9 together with any of the keys below:

\div , $-$, \times , \div , $($, $)$, $=$

You cannot put numbers together to make 2- or 3-digit numbers.
 You can use each number only once.
 The first player to get his/her calculator display to show 1 scores five points.
 If after an agreed time limit no player has reached 1, the player who is closest scores two points.

While working on this activity, students should develop their skills of mental mathematics and their fluency with numerical calculations.

Using the calculator

Calculator functions used: Subtraction, division, last answer memory

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Suppose the random number you generate is 567.

Example A:

ON/C 567 \div 9 $=$

\div 7 $=$

$-$ 8 $=$

567 \div 9 = 63.

ANS \div 7 = 9.

ANS - 8 = 1.

The answer is 1 and the game is finished.

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example B:

567 ÷ 7 =

− 9 =

÷ 8 =

You want to subtract 8 from 9, but you cannot since you have already used 8 once.
So...

÷ 3 =

− 2 =

The calculator displays 1 and the game is finished.

567 ÷ 7 = ^{DEG} 81.

ANS-9 = ^{DEG} 72.

ANS ÷ 8 = ^{DEG} 9.

ANS ÷ 3 = ^{DEG} 3.

ANS-2 = ^{DEG} 1.

..... Using the activity in the classroom

Students should be encouraged to estimate the results of calculations and think about the appropriate operations and numbers to use during the game. Let's start with 864, for example. This number is divisible by 9, 6, 3 and 2. The equation $846 \div 9$ could therefore be a possible first step. This will prompt students to test the divisibility of numbers. Students should also be encouraged to think about the various strategies they use.

The game could be played between small group of students.

..... Points for students to discuss

For some students, it may be more appropriate to start with a 2-digit number. In this case, the calculator should be set to fixed decimal place mode by pressing the [2ndF] key once and then pressing the [.] key, which has FSE written in yellow above it, until FIX is displayed at the top of the calculator screen. And press [2ndF] [TAB] and [0] keys. Doing this will round answers to 0 decimal places. The starting number can then be generated by multiplying a random number by 100.

Further Ideas

- Play the game using decimal starting numbers.
- Give the students a shuffled set of cards numbered from 1 to 9 and a shuffled set of cards numbered 10, 20, 30, 40, 50. Students choose five cards from the first set, and two cards from the second set. The calculator is then used to generate a random three digit integer, and the students have to make this total by using the numbers on the cards.

Reverse the Order

Junior high school

Objective

Develop a variety of mental methods of computation.
Estimate and approximate solutions to problems.

Explanation of the activity

Enter any 2-digit number into the calculator.
Reverse the order of the digits through simple calculator operations.

While working on this activity, students should develop their skills of mental mathematics.
They should also be interpreting and generalizing their answers.

Using the calculator

Calculator functions used: Addition, subtraction

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example A:

To reverse the order of 58:

85 - 58 =

58 + 27 =

85-58= ^{DEG} 27.

58+27= ^{DEG} 85.

Solution: Add 27 to 58 to get 85.

Now try using a 3-digit number.

Example B:

Enter 432 into the calculator

ON/C 432 - 234 =

234 + 198 =

432+234= ^{DEG} 198.

234+198= ^{DEG} 432.

Solution: Add 198 to 234 to get 432.

••••• Using the activity in the classroom •••••

This activity is probably best introduced orally to a group of students. Ask the students to enter any two digit number into their calculators. Then, ask them to find a simple way to reverse the order of the digits of these numbers. Students may do this by using inverse operations.

••••• Points for students to discuss •••••

After trying an example, the students can talk about the operations and numbers that they used. This discussion should lead to the generalization that one way to reverse the order of the digits is to add or subtract a multiple of 9. More able students could be asked to try and prove this generalization:

$$(10a + b) + N = (10b + a)$$

$$N = (10b + a) - (10a + b)$$

$$N = 9b - 9a = 9(b - a)$$

Further Ideas

Try using the activity with 3-digit numbers, 4-digit numbers, etc.

Choose any 2-digit number, reverse it, and then add the reversed number to the original.

What happens? Try this with 3-digit numbers or 4-digit numbers, etc.

Objective

Estimate and approximate solutions to problems.

Explanation of the activity

Have the class make up multiplication problems using the digits 1, 2, 3 and 4. Each digit can only be used once. Find out what the largest product among the possible answers will be.

While working on this activity, students should practice their skills of mental estimation. They should also be interpreting and generalizing their answers.

Using the calculator

Calculator functions used: Multiplication


Press the following buttons and then start operation.

ON/C 2ndF MODE 0


What is the largest number you can make by pressing the keys 1 2 3 and 4 once and only once?

Example:

$$12 \times 34 =$$

A calculator display showing the calculation 12X34= with the result 408. The display is in DEG mode.

$$2 \times 341 =$$

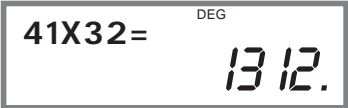
A calculator display showing the calculation 2X341= with the result 682. The display is in DEG mode.

Can you make a larger number?

Using algebra, for any four digits a, b, c, d , where $a < b < c < d$, the largest product is given by: $(10d + a) \times (10c + b)$.

Ans: The largest product is given by

$$41 \times 32 =$$

A calculator display showing the calculation 41X32= with the result 1312. The display is in DEG mode.

••••• Using the activity in the classroom •••••

This activity could be introduced to the whole class by asking students to individually make up any multiplication using only the digits 1, 2, 3 and 4. The different multiplication problems and their answers can then be compared and students can be set the task of finding the largest product. Students should be encouraged to estimate the answers to the various multiplication problems.

••••• Points for students to discuss •••••

Students can explore other sets of four numbers, generalizing the rule to find the largest product using words or symbols. After generalizing, explain the rule that for any four digits a, b, c, d , where $a < b < c < d$, the largest product is given by:

$$(10d + a) \times (10c + b).$$

If the investigation is extended to the five digits 1, 2, 3, 4, 5, then the largest product is given by:

$$431 \times 52 = 22412.$$

For some students it may be appropriate to begin with only three digits.

Further Ideas

- Find the largest product for any number of digits.
- Find the smallest product for any number of digits.
- Find the different sums that can be made by adding the digits 1, 2 and 3 once and only once. For example $12 + 3 = 15$. What happens for other sets of 3-digit numbers?

Objective

Calculate with decimals and understand the results.

Select suitable sequences of operations and methods of computation, including trial-and-improvement methods, to solve problems involving integers and decimals.

Explanation of the activity

Choose two numbers whose sum is 10.

Find out what the product of those two numbers would be.

Find the products of other pairs of numbers whose sum is 10.

Find out which number pair gives the largest possible product.

This activity helps to reinforce students' understanding of the mathematical terms 'sum' and 'product' and develops trial-and-improvement methods.

Using the calculator

Calculator functions used: Addition, multiplication, subtraction, parentheses

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Try to find the largest product of any two numbers whose sum is 10.

Example:

$$2 + 8 =$$

$$2 \times 8 =$$

You can also calculate this as $2 \times (10 - 2) = 16$.

$$2 \times (10 - 2) =$$

$$2 + 8 = 10.$$

$$2 \times 8 = 16.$$

$$2 \times (10 - 2) = 16.$$

What two numbers give the largest product?

Try multiplying various combinations of numbers whose sum is 10.

Ans: $5 \times 5 = 25$

••••• Using the activity in the classroom •••••

This activity could be introduced orally.

The largest product is 25, given by 5×5 . Some students may need to be encouraged to consider decimal numbers to verify that the largest product is 25. More able students should be encouraged to try and prove that this is the largest product.

One method of using the calculator is to enter the product as two numbers that can be edited.

Some students may prefer to enter the product as an expression such as $2 \times (10 - 2)$, which can be edited.

••••• Points for students to discuss •••••

Students could be encouraged to devise similar problems to give to each other involving numbers with different sums.

Further Ideas

- Investigate products of 3, 4, 5... numbers which have the same sum. This could be explored graphically.
(Generally, for two numbers whose sum is n , the largest product is given by $(n/2)^2$, for three numbers whose sum is n , the largest product is given by $(n/3)^3$... The nearest integer to (n/e) where $e = 2.718$ is the number of numbers which will give the maximum product.)
- The problem of finding two numbers whose product is a given total can be turned into a game where students score points according to the number of trials they perform to identify the solution. For example: The sum of two numbers is 10 and their product is 19.71. What are the two numbers?

Ans: The two numbers whose product is 19.71 are 7.3 and 2.7.

Objective

Understand and use the concept of place value in whole numbers and decimals, relating this to computation.

Calculate with decimals and understand the results; e.g. multiplying by numbers between 0 and 1. Mentally estimate and approximate solutions to numerical calculations.

Explanation of the activity

A game for two players.

- Player 1 enters any 2-digit number into the calculator.
- Player 2 then multiplies this by another number so that the answer is as close as possible to 100.
- Players score points according to how close they are to 100:
 - within 10 = 1 point
 - within 5 = 2 points
 - within 1 = 5 points
 - exactly 100 = 10 points
- Player 2 then enters a number and the game continues.
- The first player to score 20 points wins.

While working on this activity, students will be extending their understanding of decimals and improving their estimation skills.

Using the calculator

Calculator functions used: Multiplication

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example:

Player 1 enters 28.

28

DEG
28.

Player 2 multiplies this by 3.5.

× 3.5 =

28X3.5= DEG
98.

Player 2 scores two points.

The game continues until one player reaches 20 points.

••••• Using the activity in the classroom •••••

This activity could be given to students with little introduction from the teacher. Alternatively, the game could initially be played between the teacher and a large group of students. It is important that students are encouraged to think carefully about the numbers they choose and that the teacher focuses on the students' mental skills. Most benefit is obtained from the activity when students are playing together in small teams, discussing their choices of a number to multiply by.

••••• Points for students to discuss •••••

At the end of the activity, students' strategies should be discussed and compared.

Further Ideas

- Play the game with different target numbers. For example, students could multiply or divide a random number to reach a target of 1.
- The first player multiplies a random number to aim for a target of 100. The second player then multiplies this answer to try and get even closer to 100. The player who gets the calculator to display a number between 99 and 101 wins.

Ordering Fractions

Junior high school/
Elementary school
(upper grades)

Objective

Understand and use fractions.

Understand the interrelationship between fractions and decimals.

Explanation of the activity

Estimate where a given fraction would be located on a numerical line.

Check the answer using the calculator.

While working on this activity, students will be developing their understanding of the relative sizes of common fractions. The activity suggests an approach to teaching equivalence of common fractions.

Using the calculator

Calculator functions used: Addition, division, fractional calculation

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

You will need a 0 – 2 number line.

Estimate where the following common fractions should be placed on the number line and then record estimates.

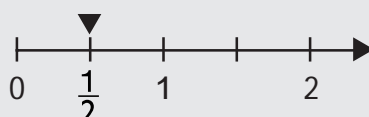
$\frac{1}{2}, \frac{15}{8}, 1\frac{2}{5}, \frac{4}{6}, \frac{3}{4}, \frac{4}{3}, \frac{14}{10}, \frac{2}{3}, 1\frac{5}{6}, \frac{3}{5}$

Use a calculator and a ruler to check your estimates.

Example A:

Find the value of the fraction $\frac{1}{2}$.

Using division:



1 ÷ 2 =

1 ÷ 2 = DEG

0.5

Using fractional calculation:

1 $\frac{1}{2}$ on the calculator display means $\frac{1}{2}$.

ON/C 1 ab% 2 =

1 $\frac{1}{2}$ = DEG

1r2.

Convert $\frac{1}{2}$ to decimal notation.

ab%

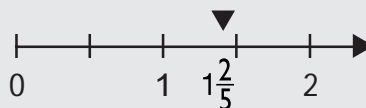
1 $\frac{1}{2}$ = DEG

0.5

Example B:

Try the fraction $1\frac{2}{5}$.

Using division:



ON/C 1 + 2 ÷ 5 =

1 + 2 ÷ 5 = DEG

1.4

Using fractional calculation:

To input the fraction $1\frac{2}{5}$:

1 $\frac{2}{5}$ on the calculator display means $1\frac{2}{5}$.

Convert $1\frac{2}{5}$ to decimal notation.

ON/C 1 ab% 2 ab% 5 =

1 $\frac{2}{5}$ = DEG

1r2r5.

Ordering Fractions

Junior high school/
Elementary school
(upper grades)

Convert to a common fraction.

$\frac{a}{b}\%$

Try checking these decimal fractions against the estimates you put on the number line.

$\frac{2ndF}{d/c}$

Example C:

Try $\frac{4}{8}$.

Using division:

$\frac{ON/C}{4} \div 8 =$

Convert to a fraction.

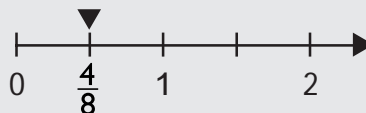
Using fractional calculation, input $\frac{4}{8}$.

$\frac{ON/C}{4} \frac{a/b\%}{8} =$

You can see that $\frac{4}{8} = \frac{1}{2}$.

Convert $\frac{4}{8}$ to decimal notation.

$\frac{a/b\%}{}$



$1 \div 2 =$ DEG
14

$1 \div 2 =$ DEG
75.

$4 \div 8 =$ DEG
0.5

$4 \div 8 =$ DEG
12.

$4 \div 8 =$ DEG
0.5

..... Using the activity in the classroom

This activity may be introduced orally. The number line could be copied onto an overhead projector transparency or written on the board. Divide students into small groups and give each group a fraction card. Have the groups discuss where to place their given fraction on the line. Groups then take turns marking their fractions on the number line. Solutions can be discussed, together with methods of checking the solutions. This should lead into converting common fractions to decimal notation, and students can be shown how to do this on the calculator. It is important that students are aware of the general method of converting common fractions into decimal notation (dividing the numerator by the denominator), as well as the use of the fraction key on the calculator.

..... Points for students to discuss

It will be discovered that some of the fractions are equivalent to each other and this leads into the second part of the activity. When the fraction $\frac{4}{6}$ is entered into the calculator, pressing '=' simplifies the fraction to $\frac{2}{3}$. Students should explore the results of entering different fractions, thus generating sets of equivalent fractions. It is important that students are encouraged to understand the concept of equivalence.

Further Ideas

Small groups of students are given a pack of cards with a different fraction on each card. The group has to sort the cards into sets, so that all the cards in each set are equivalent to each other.

Adding Fractions

Junior high school/
Elementary school
(upper grades)

Objective

Understand and use fractions.

Calculate with fractions and understand the results.

Explanation of the activity

Using the calculator, find the sum of two given fractions each having 1 in the numerator.

Look for patterns to help understand how to add the fractions without using the calculator.

This activity suggests an approach to teaching addition of common fractions.

Using the calculator

Calculator functions used: Addition, fractional calculation

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example:

Using fractional calculation, find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

1 $\frac{a}{b}\%$ 2 + 1 $\frac{a}{b}\%$ 3 =

$\frac{1}{2} + \frac{1}{3} =$ on the calculator display means $\frac{5}{6}$.

Convert $\frac{5}{6}$ to decimal notation.

$\frac{a}{b}\%$

0.83333... $\frac{a}{b}\%$ on the calculator display means $\frac{5}{6}$.

Find the sums of other common fractions.

1 $\frac{a}{b}\%$ 5 + 1 $\frac{a}{b}\%$ 7 =

$\frac{a}{b}\%$

1 $\frac{a}{b}\%$ 2 + 1 $\frac{a}{b}\%$ 3 =
5 $\frac{a}{b}\%$

1 $\frac{a}{b}\%$ 2 + 1 $\frac{a}{b}\%$ 3 =
0.833333333

1 $\frac{a}{b}\%$ 5 + 1 $\frac{a}{b}\%$ 7 =
12 $\frac{a}{b}\%$ 35

1 $\frac{a}{b}\%$ 5 + 1 $\frac{a}{b}\%$ 7 =
0.342857142

Adding Fractions

Junior high school/
Elementary school
(upper grades)

••••• Using the activity in the classroom •••••

This activity should be presented after studying equivalence of common fractions.

The activity is best introduced orally. Ensure that the students know how to add two common fractions on the calculator. Ask them to add $\frac{1}{2}$ and $\frac{1}{3}$ and record the answer ($\frac{5}{6}$). Ask the students if they can see any connection between the answer and the original two fractions. Students may note that $2 + 3 = 5$ and $2 \times 3 = 6$. Allow students to explore other unit fractions and encourage them to generalize. Students should be asked to try and explain what is happening. It should be noted that the pattern may appear to break down when fractions with a common denominator are added.

••••• Points for students to discuss •••••

Students can then explore what happens when other common fractions are added. For some students, it may be appropriate to begin by considering a pair of fractions that includes one unit fraction.

It is important that students are encouraged to understand what is happening, and that reference is made to equivalent fractions.

Further Ideas

- Investigate subtracting, multiplying or dividing common fractions.
- The Babylonians mostly used fractions which had 1 as the numerator. For example, $\frac{5}{6}$ could be written as $\frac{1}{2} + \frac{1}{3}$. Investigate Babylonian fractions.

Halfway Between

Junior high school/
Elementary school
(upper grades)

Objective

Understand and use fractions.

Calculate with fractions and understand the results.

Explanation of the activity

Use the calculator to find the fraction that is exactly halfway between two other fractions.

Look for patterns to help understand how to find the answer without using the calculator.

This activity reinforces addition of common fractions and considers the result of dividing common fractions by integers. By working on the activity, students should also develop an increasing feel for the relative sizes of fractions.

Using the calculator

Calculator functions used: Addition, division, multiplication, fraction, calculation

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Example A:

Find the fraction that is halfway between $\frac{1}{2}$ and $\frac{1}{3}$.

Using fractional calculation, obtain the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

1 $\frac{a}{b}$ 2 + 1 $\frac{a}{b}$ 3 =

1 $\frac{a}{b}$ 2 + 1 $\frac{a}{b}$ 3 = $5r6.$
<Display 1>

Half of this fraction is the number you are looking for, so divide this fraction by 2.

\div 2 =

ANS \div 2 = $5r12.$

Or after <Display 1>, multiply by $\frac{1}{2}$.

\times 1 $\frac{a}{b}$ 2 =

ANS \times 1 $\frac{a}{b}$ 2 = $5r24.$

Example B:

Find the fraction that is halfway between $\frac{1}{3}$ and $\frac{1}{4}$.

Using fractional calculation, obtain the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

1 $\frac{a}{b}$ 3 + 1 $\frac{a}{b}$ 4 =

1 $\frac{a}{b}$ 3 + 1 $\frac{a}{b}$ 4 = $7r12.$
<Display 2>

Halfway Between

Junior high school/
Elementary school
(upper grades)

Half of this fraction is the number you are looking for,
so divide this fraction by 2.

$$\div 2 =$$

Or after <Display 2>, multiply by $\frac{1}{2}$.

$$\times 1 \frac{a}{b} 2 =$$

$$\text{ANS} \div 2 = \overset{\text{DEG}}{7.24.}$$

$$\text{ANS} \times 1 \frac{1}{2} = \overset{\text{DEG}}{7.24.}$$

Continue the activity using other common fractions.

..... Using the activity in the classroom

This activity could follow the study of addition of common fractions.

The activity is best introduced orally. Ask the students to give different fractions that lie between $\frac{5}{12}$ and $\frac{2}{3}$. One possibility is to arrange these on a fraction line. It is important that students are challenged to justify their answers and, in some cases, it may be appropriate to consider decimal equivalents. The students should then be asked to identify the common fraction that is halfway between $\frac{5}{12}$ and $\frac{8}{12}$, justifying their answer.

..... Points for students to discuss

Furthering the activity, students can be asked to give fractions that lie between $\frac{1}{2}$ and $\frac{1}{3}$ and identify the common fraction that is halfway between them. At this stage it may be necessary to discuss methods for finding a number that is halfway between two numbers. Students can then use their calculators to identify fractions that are halfway between other unit fractions. This can be extended to non-unit fractions. It is important that students are encouraged to understand what is happening.

Further Ideas

- Find fractions that lie $\frac{1}{3}$ of the way between two fractions, or $\frac{1}{4}$ of the way between two fractions, etc.

Objective

Understand and use the concept of place value in decimals.

Understand and use decimals and fractions while comprehending the interrelationship between them.

Use some common properties of numbers, including multiples.

Give solutions in the context of the problem, selecting the appropriate degree of accuracy and interpreting the display on a calculator.

Explanation of the activity

Use the calculator to find fractions that are near integers in decimal form.

While working on this activity, students will be developing their understanding of decimals, particularly their relationship with fractions.

Using the calculator

Calculator functions used: Multiplication, division

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

The 35th multiple of 0.314 is 10.99.

$$35 \times 0.314 = 10.99$$

35 \times 0.314 =

35X0.314= 10.99

10.99 is a 'near integer'; it is nearly 11.

Using fractional calculation, input $\frac{11}{35}$.

ON/C 11 ab% 35 =

11r35= 11r35.

Convert $\frac{11}{35}$ to decimal notation.

ab%

11r35= 0.314285714

Using division, divide 11 by 35.

The fraction $\frac{11}{35}$ has a decimal value close to 0.314.

11 \div 35 =

11÷35= 0.314285714

••••• Using the activity in the classroom •••••

This activity is probably best introduced orally. Students could use the sequence function of the calculator to generate the multiples of some integers, and could then begin to investigate the multiples of some decimals.

••••• Points for students to discuss •••••

The teacher could ask the students to generate the multiples of 0.314, challenging them to find a multiple that is nearly an integer. Students can then begin to investigate the situation further.

Further Ideas

- Use this idea to investigate different approximations for π . For example, $22/7 = 3.142857$, whereas $\pi \approx 3.141593$. However, $179/57 = 3.140351$.
- Investigate approximations for $\sqrt{2}$, or $\sqrt{3}$, etc.

Objective

Reinforce students' understanding of the equivalence of shapes in various alignments and how this relates to multiplication within a practical context.

Develop mental skills involving factors, divisors, and systematic thinking.

Explanation of the activity

12 cubes, each with a volume of 1 cm^3 , may be placed together to create any of four cuboids, each having a volume of 12 cm^3 .

Find the equivalent equations for each of the cuboids; for example,

$1 \times 1 \times 12 = 12$, $2 \times 2 \times 3 = 12$, etc.

Using the calculator

Calculator functions used: Multiplication, Multi-line Playback

Press the following buttons and then start operation.

ON/C **2ndF** **MODE** **0**

Introduce students to the calculator's Multi-line playback feature, which will be useful to display sets of solutions for each "volume" number.

1 **×** 1 **×** 12 **=**

2 **×** 2 **×** 3 **=**

1 **×** 3 **×** 4 **=** etc.

1X1X12= 12.

2X2X3= 12.

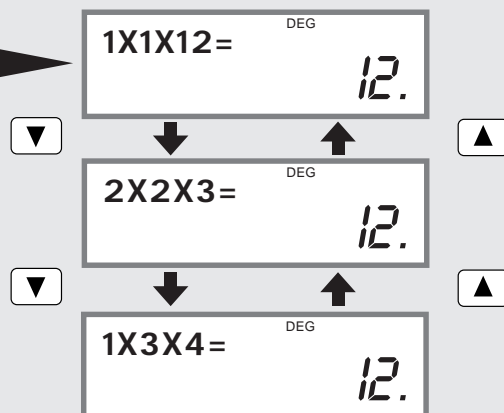
1X3X4= 12.

Each press of a **▲** **▼** key takes you one calculation step forward or backward.

Display the first calculation

with **2ndF** **▲**

2ndF **▲**



Find the five calculations that represent cuboids that each have a volume of 30 cm^3 .

e.g. $1 \times 1 \times 30 =$ etc.

In a similar way, find the twelve calculations for cuboids each having a volume of 96 cm^3 .

How many similar calculations must there be for 180 cm^3 ?

Which of these cuboids is nearest to looking like a cube?

For volumes between 150 cm^3 and 200 cm^3 , which particular ones can be represented by at least 16 cuboids each? Which volumes have the smallest number of cuboids each?



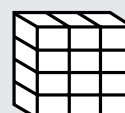
$$1 \times 1 \times 12 = 12$$



$$1 \times 2 \times 6 = 12$$



$$2 \times 2 \times 3 = 12$$



$$1 \times 3 \times 4 = 12$$

••••• Using the activity in the classroom •••••

Students may benefit from the use of actual blocks that can be stacked to form the different cubic combinations. An OHP calculator could also be used to collect solutions from the entire class.

••••• Points for students to discuss •••••

The number of divisors for a number expressed as $p^a \times q^b \times r^c$ (where p, q , and r are all prime) is $(a + 1)(b + 1)(c + 1)$. For example, $360 = 2^3 \times 3^2 \times 5^1$. Here, $a = 3, b = 2$, and $c = 1$, so the number of divisors is given by the expression $(3 + 1)(2 + 1)(1 + 1) = 24$. Therefore, 360 has 24 divisors.

Further Ideas

- Use trial and improvement to find the side of a cube having a volume of 180 cm^3 .
- Move into "four (or more) dimensions" as a means of finding the factors of a number. For example, $6006 = 77 \times 78 = (7 \times 11) \times (6 \times 13) = 2 \times 3 \times 7 \times 11 \times 13$. All stages can be displayed using the replay function.

Function Tables

Junior high school

Objective

Understand and use calculator functions.
Understand and apply functional relationships.
Enable speedy plotting of graphs.

Explanation of the activity

Use the calculator to calculate the y values for a given function using a set range of values for x .
Record the values on a table and use them to plot a graph.

Using the calculator

Calculator functions used: Multiplication, editing, Multi-line Playback

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

x	-5	-4	-3	-2	-1	...	5
y	-25	-21	-17	-13	-9	...	15

Enter the y values for the function $y = 4x - 5$ using the values from -5 to +5 for x .
Use the calculator's playback function to calculate the functions efficiently. After calculating the values, use them to plot the graph of $y = 4x - 5$.

4 \times 5 \div $-$ 5 $=$

\blacktriangleright \blacktriangleright \blacktriangleright \blacktriangleright \blacktriangleright

4 DEL

$=$

$4X(-5)-5=$ DEG $-25.$

$4X(-5)-5=$ DEG

$4X(-4)-5=$ DEG

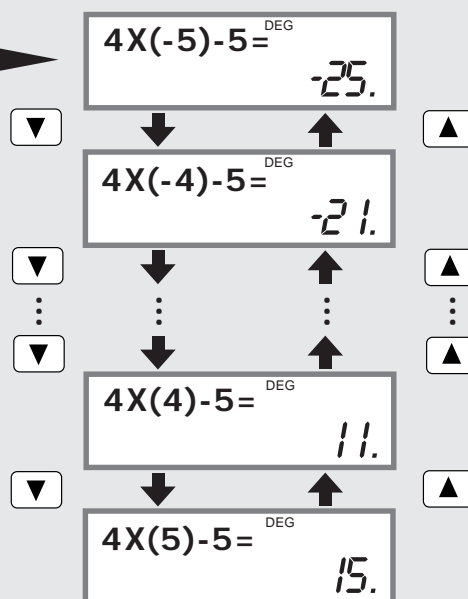
$4X(-4)-5=$ DEG $-21.$

2ndF \blacktriangle

In the same way, find the values for
 $x = -3, -2, 1, 0, \dots, 5$.

Each press of a \blacktriangle \blacktriangledown key takes you
one calculation step forward or backward.

Display the first calculation with 2ndF \blacktriangle



Return to the calculation for $x = 5$ and redo the calculation for the equation $y = 4x - 3$.
Add another line to the table and calculate the values of y for the new equation.
Plot the second graph with the first on the same axis.

What do you notice about the graphs and the numbers in the table?

What do you think will happen if you try another similar equation such as $y = 4x - 1$,
 $y = 4x + 1$, or $y = 4x + 4$?

Can you explain the number pattern and the picture you have produced?

••••• Using the activity in the classroom •••••

This activity should be introduced after practicing substitution.

Start the activity as a whole class so the students can gain confidence in using the calculator and see the advantages of calculating first and then recording the results to speed up the process of making the graph table. The students can calculate the y values for the second equation themselves and quickly continue with other suggested equations using multi-line playback to go directly from the (x, y) values to the graph without needing to record the result in a table. This enables the families of graphs to be compared rapidly. Try extending the activity by using graphs with different gradients to establish the parallel nature of the graphs, and then try keeping the intercept constant and varying the gradient.

••••• Points for students to discuss •••••

The idea of using the playback function as a rapid way to calculate function values can be applied to a wide range of equations including polynomials, trigonometric functions, etc. Students can do calculations in one sequence and then use the playback function to go back through the answers and record or plot them all at once.

Further Ideas

Investigations on graphs can be done more quickly if the playback function is used so each function does not have to be retyped at every entry. Demonstrate this by using the following suggestions:

- Solve a quadratic function such as $ax^2 + bx + c = 0$ for varying values of a , b , and c .
- Use the calculator to generate values of a trigonometric function and enter the results directly onto a graph using the playback function.

Objective

Understand and use the concept of place value in whole numbers.
Explore a variety of situations that lead to the expression of relationships.
Construct and interpret formulas and expressions.
Manipulate algebraic expressions.

Explanation of the activity

A word that reads the same forwards and backwards, such as "mom" and "level", is called a palindrome. A palindromic number is exactly the same; the number has the same value whichever way you write the digits. For Example, 212, 34543 and 10001.

Using the calculator

Calculator functions used: Division

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Write down some 4-digit palindromes and use the calculator to divide each of them by 11.

Example:

ON/C 2332 ÷ 11 =

5665 ÷ 11 = etc.

2332 ÷ 11 = ^{DEG} 212.

5665 ÷ K = ^{DEG} 515.

Do you notice a pattern?

The use of the playback function will speed up the calculations and enable students to compare results to look for a pattern.

Make up some more 4-digit palindromes and divide each of them by 11.
Have students compare their results until they notice a pattern.

••••• Using the activity in the classroom •••••

This is an activity that can first be given to students to work on and the results later discussed as a group. The object is to discover a pattern in the results—the pattern being that the answer will always be a whole number.

••••• Points for students to discuss •••••

The pattern may be too obvious for the students to mention, so it may be necessary to give them the following hint:

Try writing the number in its long form; for example,

$$2332 = 2 \times 1000 + 3 \times 100 + 3 \times 10 + 2$$

Further Ideas

- Try to explain the problem using algebra. You could start off by giving the class just the first line of the calculation below and let them work on the rest individually or in groups.

$$\begin{aligned} a b b a &= a \times 1000 + b \times 100 + b \times 10 + a \\ &= a \times 1000 + a + b \times 100 + b \times 10 \\ &= a (1000 + 1) + b (100 + 10) \\ &= a \times 1001 + b \times 110 \\ &= a 91 \times 11 + b \times 10 \times 11 \\ &= 11 (91a + 10b) \end{aligned}$$

- Try the activity using 6-digit palindromes.
Have the class prove that not all 5-digit palindromes are exactly divisible by 11.

Trial and Improvement

Junior high school

Objective

Mentally estimate and approximate solutions to numerical calculations.

Understand and use the concept of place value in decimals and relate it to computation.

Explanation of the activity

Use “trial and improvement” to find the length of the side of a cube-shaped box that can hold 100 cm³ of ice cream.

The two mental calculations $4 \times 4 \times 4 = 64$ and $5 \times 5 \times 5 = 125$ should suggest a possible starting calculation such as $4.5 \times 4.5 \times 4.5 = 91$, which can be shortened to $4.5^3 = 91$.

This activity gives students the opportunity to enhance their understanding of decimals and improve their skills in estimation.

Using the calculator

Calculator functions used: Multiplication, FSE, TAB

Press the following buttons and then start operation.

ON/C **2ndF** **MODE** **0**

Set the calculator to “fixed point” notation with a TAB value of 0.

(Doing this will display answers to the nearest whole number.)

Adjust the TAB setting to 1 and then continue to improve the accuracy of the answer

2ndF **FSE** **2ndF** **TAB** 1

4 **×** 4 **×** 4 **=**

5 **×** 5 **×** 5 **=**

4.9 **×** 4.9 **×** 4.9 **=**

⋮

4.7 **×** 4.7 **×** 4.7 **=**

4.6 **×** 4.6 **×** 4.6 **=**

From this we can see the answer lies between 4.6 and 4.7. Continue to search for the answer repeating this operation.

DEG
0.

FIX DEG
0.0

FIX DEG
4X4X4= 64.0

FIX DEG
5X5X5= 125.0

FIX DEG
4.9X4.9X4.9= 117.6

⋮

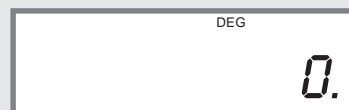
FIX DEG
4.7X4.7X4.7= 103.8

FIX DEG
4.6X4.6X4.6= 97.3

FIX DEG
4.642X4.642X → 100.0

Switch FSE and TAB to normal display for further operation.

ON/C 2ndF FSE 2ndF FSE 2ndF FSE



Press FSE until FIX, SCI, or ENG are not shown on the display.

..... Using the activity in the classroom

This activity may be given to students with little introduction or, with the use of the OHP unit, this or a similar task may be introduced to the whole class followed by individual work on one or more of the extension activities. The use of the multi-line playback function will be of practical benefit in tackling questions involving trial and improvement.

..... Points for students to discuss

It will be necessary to familiarize the students with the FSE and TAB keys in order to understand, for example, why 4.641^3 and 4.642^3 both have the value 100 to the nearest unit. In the context of similar problems, students will need to consider what degrees of accuracy are appropriate; in the case of cubic centimeters of ice cream, possibly only to one decimal place.

Further Ideas

- Find the side of a cubical carton whose volume is $\frac{1}{2}$ liter. It may be necessary to remind students of the equivalence of 500 ml (fluid measure) and 500 cm^3 (solid measure).
- Find the dimensions of a fruit juice carton whose sides are in the proportion 1 : 2 : 3 and whose capacity is 1 liter.
- Find the Golden Ratio x by trial and improvement of the relation

$$\text{Guess } x (\text{Guess} + 1) = 1$$

Use the playback function on the calculator to show that

$$x = 1 / (1 + x) \text{ and that } x = \sqrt{1 - x}.$$

All metric paper has the same shape (except golden). If A0 has an area of 1 m^2 and the longer side is $\sqrt{2}$ times bigger than the smaller side, find these dimensions. What are the dimensions of A4? Have the students confirm their calculations by measuring a sheet.

Objective

Use last digits as a means of checking the output of a calculator.

Practice estimation and observe patterns.

Reinforce the concept of prime numbers.

Explanation of the activity

Perform a series of multiplication equations keeping the last digit of each of the multipliers constant; for example, 3×7 , 13×7 , 3×17 , etc.

Using the calculator

Calculator functions used: Multiplication

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

Enter the following equations into the calculator:

$$3 \times 7 =$$

$$13 \times 7 =$$

$$3 \times 17 =$$

$$3 \times 7 = 21$$

$$13 \times 7 = 91$$

$$3 \times 17 = 51$$

Find other last digit combinations that give answers ending in 1.

Which of the numbers in the following set can be made from the product of two numbers? (excluding equations using 1 multiplied by the number itself)

21, 41, 51, 61, 71, 81, 91, 101, 111, 121, 131, 141, 151, 161, 171, 181, 191, 201

Which of the numbers can be made in more than one way?

Make a collection of your calculations so that they can be displayed in order of answer size.

Name the type of numbers that cannot be made.

••••• Using the activity in the classroom •••••

It is probably best to first introduce the activity as a class to give the students an opportunity to make estimates before using their calculators. Once the class has shared their initial ideas, they can be given time to investigate any patterns they discover.

••••• Points for students to discuss •••••

After investigating patterns on their own, students should share their discoveries with the rest of the class.

Further Ideas

- Examine the first 20 prime numbers. Except for the number 2, they all end with an odd last digit. Repeat the procedure for the last digits of 3, 7, and 9. Find all the prime numbers between 1 and 201.
- Find the last digits to:
 1. the answers to the multiplication tables.
 2. the square numbers.
 3. other number sequences such as the cube numbers and triangle numbers.
- Find the two consecutive numbers whose product is 6006.

A Question of Interest

Junior high school

Objective

Understand, use and calculate with percentages.

Select suitable sequences of operations and methods of computation, including trial-and-improvement methods, to solve problems involving integers, decimals and percentages.

Give solutions in the context of the problem, selecting an appropriate degree of accuracy, and interpret the display on a calculator.

Explanation of the activity

Use the calculator to find solutions to problems involving interest rates.

While working on this activity, students will develop their understanding of percentages within the context of compound interest situations.

Using the calculator

Calculator functions used: % calculation, multiplication

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

If you invest money at a certain level of interest, by how much will your money grow?

Example:

\$100 is invested at 0.1% annual interest.

Using multiplication: Multiply the principal \$100 by 1.001.

100 \times 1.001 =

100X1.001^{DEG}=
100.1

For the total after two years, multiply the previous answer again by 1.001.

\times 1.001 =

ANSX1.001^{DEG}=
100.2001

After three years... =

ANSX1.001^{DEG}=
100.3003001

After four years... =

ANSX1.001^{DEG}=
100.4006004

After 10 years, you have approximately \$101.

= = = = = =

ANSX1.001^{DEG}=
101.004512

A Question of Interest

Junior high school

Using the % calculation key: After one year, you should have 0.1% of your \$100.

100 $\boxed{+}$ 0.1 $\boxed{2ndF}$ $\boxed{\%}$

100+0.1%^{DEG}
100.1

You now have \$100.10.

After two years, you have 0.1% more. $\boxed{+}$ 0.1 $\boxed{2ndF}$ $\boxed{\%}$

ANS+0.1%^{DEG}
100.2001

After three years... $\boxed{2ndF}$ $\boxed{\%}$

ANS+0.1%^{DEG}
100.3003001

After 10 years...

$\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$ $\boxed{2ndF}$ $\boxed{\%}$

You have approximately \$101.

ANS+0.1%^{DEG}
101.004512

..... Using the activity in the classroom

This activity is probably best introduced orally. After a discussion about investments and interest rates, the teacher can use the sequence function of the calculator to generate sequences showing how an initial capital sum grows for a fixed interest rate. Students can be asked to find the annual interest rate that ensures their money is doubled in 10 years.

Students can then investigate the annual interest rates that would double their money for different numbers of years. These interest rates could be plotted on a graph.

5 years	14.9%
10 years	7.2%
15 years	4.7%
20 years	3.6%
25 years	2.8%

..... Points for students to discuss

It may be useful to show students how to generate sequences on the calculator.

Further Ideas

- Investigate interest rates that would triple an investment, or...
- From 1970 to 1980 prices tripled. What was the average rate of inflation?

A Question of Interest

Junior high school

For High school Students

How much will your investment be worth in n years?
Let's make an equation.

The original amount of money invested, called the principal, multiplies each year by the amount x .

Let's use this equation to see how much money we have after 100 years.

Press the following buttons and then start operation.

ON/C 2ndF MODE 0

The original amount, or principal, is \$100; so $a = 100$.

The number of years is 100; so $n = 100$.

The interest is 0.1%; so $x = 1.001$.

100 \times 1.001 y^x 100 $=$

100X1.001^{DEG}100 →
110.5115698

You have \$110.50 after 100 years.

How many years would it take for the money to double?

Let's make an equation.

The money invested multiplies each year by the amount x .

After n years the money doubles, so...

Divide both sides by ' a '

Calculates the 'log' of both sides

$$ax^n = 2a$$

$$x^n = 2$$

$$\log x^n = \log 2$$

$$n \log x = \log 2$$

$$\therefore n = \log 2 / \log x$$

If ' a ' is the money deposited, the savings would double.

log 2 \div log 1.001 $=$

log2^{DEG}÷log1.00 →
693.4936964

It takes approximately 694 years for your money to double.

Getting Even

Elementary school
(upper grade)

Objective

Use some common properties of numbers.

Explore a variety of situations that lead to the expression of relationships.

Explanation of the activity

A game of chance to compare the relationship between odd and even numbers.

By working on this activity, students will reinforce their understanding of odd and even numbers. More able students could develop their skills in using algebra to prove generalizations.

Using the calculator

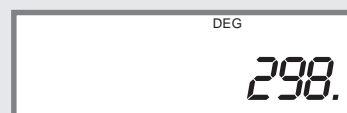
Calculator functions used: Addition

Press the following buttons and then start operation.



A game for two players

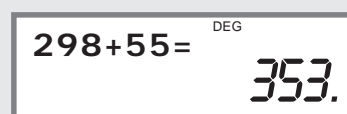
The first player enters any number into his/her calculator without showing it to the other player.
For example, 298.



The second player then enters a number into his/her calculator without showing it to the other player.
For example, 55.



The players then show each other their numbers and add them. If the answer is even the first player scores 1 point; if the answer is odd, the second player scores 1 point.



The first player to score 10 points is the winner.

..... Using the activity in the classroom

The game is best played between pairs or small groups of students. It could be introduced by the teacher playing the game against some students.

While playing the game, students should be encouraged to reflect on whether the game is fair, and also try and think about the reasons for their conjectures.

Odd + Even = Odd

Even + Odd = Odd

Odd + Odd = Even

Even + Even = Even

..... Points for students to discuss

More able students could try to formally prove their conjectures.

The idea can be extended by students thinking about the conditions for obtaining even or odd answers when three numbers are added, or four numbers, or...

Further Ideas

More able students could try to formally prove their conjectures.

The idea can be extended by students thinking about the conditions for obtaining even or odd answers when three numbers are added, or four numbers, or...

